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Factors influencing the accuracy of biomechanical breast models

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Recently it has been suggested that finite element methods could be used to predict breast deformations in a number of applications, including comparison of multimodality images, validation of image registration and image guided interventions. Unfortunately, knowledge of the mechanical properties of breast tissues is limited. This study evaluated the accuracy with which biomechanical breast models based on finite element methods can predict the displacements of tissue within the breast during the clinical situation where the boundaries of the organ might be known reasonably accurately. Breast deformations were derived from a three-dimensional nonrigid image registration. Six linear and three nonlinear elastic models with and without skin were tested. These models were compared to hyperelastic models. The accuracy of the models was evaluated by assessing the ability of the model to predict the location of 12 corresponding anatomical landmarks. The accuracy was most sensitive to the Poisson’s ratio and the boundary condition. Best results were achieved for accurate boundary conditions, appropriate Poisson’s ratios and models where fibroglandular tissue was at most four times stiffer than fatty tissue. These configurations reduced the mean (maximum) distance of the landmarks from 6.6 mm (12.4 mm) to 2.1 mm (3.4 mm) averaged over all experiments. © 2006 American Association of Physicists in Medicine. [DOI: 10.1118/1.2198315]

Key words: Finite element analysis, breast imaging, image guided interventions, deformation

I. INTRODUCTION

Biomechanical breast models employing finite element methods (FEMs) have been explored for predicting mechanical deformations during biopsy procedures, for modeling compressions similar to x-ray mammography, for the registration of magnetic resonance (MR) and x-ray mammograms, for validating the nonrigid registration of dynamic contrast-enhanced (DCE) MR mammograms, for testing reconstruction algorithms in elastography, and as a forward model for elastography. The use of mechanical models in these applications can be seen as providing prior information about the expected deformations. This prior information helps to constrain possible solutions and thus to provide a physical basis for interpolation, reconstruction and prediction for those cases in which there is insufficient information in the image data. Previously described biomechanical breast models have been assessed based on the predicted location of anatomical landmarks selected in breast images acquired before and after in vivo compression, by visual comparison of the simulated compressed breast image with the uncompressed breast, or not at all. Evaluations based on landmarks were limited due to the use of only very few landmarks or a single dataset.

The first aim of this study was to determine a suitable model configuration, that can be employed for simulating plausible breast deformations during DCE MR mammography for the validation of nonrigid registration. The second objective was to determine the most important modelling aspects. Biomechanical breast models are commonly criticized for modelling the mechanical breast properties in a too simplistic or unrealistic manner by using rather poor estimates of the biomechanical properties. Yet, the influence of the mechanical breast properties on the model accuracy in comparison to other modelling factors has not been systematically investigated for the practical clinical situation where the boundary conditions may be known rather well from imaging systems yet the mechanical properties are rather poorly characterized.

Biomechanical breast models mainly vary with respect to the mesh generation, the boundary conditions employed, the tissue properties assumed and the solution strategies. This
II. MATERIALS AND METHODS

A. Datasets

Two volunteers were recruited for this study, a 37 year old nulliparous pre-menopausal woman and a 65 year old multiparous post-menopausal woman on hormone replacement therapy. TI weighted images of the volunteers were acquired on a Philips 1.5T Intera using a 3D fast gradient echo sequence with TR=16.9, TE=6.0, flip angle 35°, axial slice direction, and a spatial resolution of 0.82×0.82×2 mm³ and 0.82×0.82×2.5 mm³, respectively. Each volunteer was positioned in a fixation device for breast biopsies provided by Philips Medical Systems. The right breast was placed between two sagittal plates without any compression for the first image. For the second image, the plate on the breast’s medial side was kept immobile, while the plate on the lateral side was moved manually towards the immobile plate by as much as the volunteer could comfortably tolerate. The images showed that this resulted in a compression of about 20% in breast thickness for each volunteer (from 75 mm to 60 mm for volunteer one and from 78 mm to 63 mm for volunteer two). Volunteers were asked not to move between these two scans. Consent of volunteers was obtained in accordance with Guy’s Hospital, London, UK, local research ethical approval 00/11/99. Figures 6(a) and 7(a) show example slices of these images.

B. Nonrigid registration

Surface displacements were derived from a full 3D nonrigid registration, rather than a 3D surface registration, to improve accuracy by avoiding segmentation and decimation errors. The nonrigid registration algorithm described in Rueckert et al. was employed to register the image of the compressed breast to the image of the uncompressed breast. The registration was not constrained to conserve volume because blood volume may have been reduced due to the compression. The images were nonrigidly registered using a regular grid of control points approximated by B-splines and normalized mutual information as a voxel based similarity measure. The registration algorithm was applied in a multi-resolution fashion, with an initial control point spacing of 40 mm. At every resolution level this distance was halved. Based on a visual assessment of the registration accuracy, the process was terminated at a final control point spacing of 10 mm for volunteer one and 2.5 mm for volunteer two. Acquisition of the images within 15 minutes, in the same setting, with a high spatial resolution and without the administration of contrast agent simplified the task of registration. Example slices of the registered images are shown in Figures 6(b) and 7(b).

C. Biomechanical breast models

1. Geometric FEM model

The 3D MR images were first corrected for intensity nonuniformity using the N3 program. The images of the uncompressed breast were then manually segmented into breast and background, using an interactive tool in ANALYZE. The breast regions were further segmented into fatty and fibroglandular tissue using thresholding followed by manual editing if necessary. The breast segmentations were blurred using an isotropic 3D Gaussian kernel with a standard deviation of 1 mm. The blurred segmentations were then downsampled to an isotropic voxel size of 7.5 mm to 10 mm (Table I) such that different mesh resolutions could be tested and the limit of available elements was not exceeded during the meshing stage. A 3D triangulation of the outer surface of the breast was obtained using marching cubes, smoothing and decimation techniques from The Visualization Toolkit.
The triangulated volumes were meshed into 10-noded tetrahedral elements with the ANSYS FEM package. These elements have an additional node on each edge, which allows for quadratic interpolation of the element properties. The skin was modelled by adding 1 mm thick triangular prisms with additional nodes in the middle of each edge on the surface of the breast. Material properties were assigned to the elements according to the image segmentation.

Two geometrical models, called GM1 and GM2, were created for each volunteer to investigate the combined influence of the element size and shape. The GM1 models had about one-half as many elements as the GM2 models, see Table I. This was achieved either by decimating the triangular surface more or by down-sampling the blurred segmentations more.

Figure 1 shows a 2D example slice of the breast image, the segmentation and the corresponding meshes for volunteer one.

2. Material models

Published values of the stress to strain relationship (Young’s modulus) of breast tissue types vary considerably. Models were constructed that covered the range of values and complexity of published stress-strain relationships in order to determine their influence on predicted deformations in a systematic and quantitative way.

(a) Linear and piecewise-linear models: The individual tissue types were modelled as isotropic and homogeneous. Only displacement boundary conditions were applied, and hence only the ratios of the Young’s modulus of the different tissue types were required. Six linear models were constructed where fibroglandular tissue was 1, 1.5, 5, 10, 15 or 20 times stiffer than fatty tissue. These values cover and increase the range of reported ex-vivo ratios (1.5 to 5.0 for up to 5% prestraining) and in-vivo ratios (1.3 to

<p>| Table II. Young’s modulus $E$ of fatty and fibroglanular tissue for linear and piecewise-linear material models. The same Young’s modulus was used for compression as for tension, i.e., $E(t)=E(e)$. The last column lists mean($E_f/E_g$) for strain $e \in [-0.25,0]$. |
|-----------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Name</th>
<th>Fatty tissue $E_f$</th>
<th>Fibroglandular tissue $E_g$</th>
<th>$E_g/E_f$</th>
</tr>
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<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MM1</td>
<td>1</td>
<td>1</td>
<td>1.0</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>MM7</td>
<td>$718.16 e^{4.46}$ if $0 \leq e &lt; 0.25$</td>
<td>$15.1 \exp(10.0 e)$</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$184$ if $e \geq 0.25$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MM8</td>
<td>18.5</td>
<td>$1001.3e^2-272.7e+86.1$</td>
<td>4.0</td>
</tr>
<tr>
<td>MM9</td>
<td>$519.7e^2+2.4e+4.9$</td>
<td>$123 888.9e^3-11 766.7e^2+696.9e+12.1$</td>
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</table>
Three nonlinear models were created in accordance to previously reported values. These nine models were tested without skin and with skin that was 10 times stiffer than the fatty tissue in the linear models. The nonlinear stress-strain functions were approximated by piecewise-linear functions. The material properties of these models are summarized in Table II. Their relative stress-strain functions are depicted in Figure 2.

(b) Hyperelastic models: Hyperelastic materials can experience large elastic strain that is recoverable and have generally very small compressibility. The stress-strain relation of hyperelastic materials is, by definition, completely defined by a strain-energy function. We assessed the neo-Hookean model and the five-parameter Mooney-Rivlin model, where \( \alpha_{ik} \) are the model constants, \( I_1 = I_1^2 + I_2^2 + I_3^2 \), and \( \lambda_i \) are the eigenvalues of the stretch tensor. These models were fitted in the least squares sense to the strain-stress relationship of MM1-MM9 for strains within \( \pm 25\% \). Parameter fitting with ANSYS led to unstable materials for the Mooney-Rivlin models. A constrained parameter fitting was developed instead. The constraints ensured that the work required for an arbitrary change in deformation is always positive. No convergence problems were experienced for models derived in this way. The parameters of these models are listed in Table III. Figure 3 shows their relative stress-strain relationships in comparison to the original functions. Neo-Hookean models approximated linear models well. Mooney-Rivlin models achieved better fits for nonlinear models due to their greater number of free parameters.

(c) Poisson’s ratios: The influence of the Poisson’s ratio was assessed for values 0.499, 0.495, 0.45, 0.4, 0.3, 0.2, and 0.1 for the linear and piecewise-linear material models.

### 3. Boundary conditions

The influence of boundary conditions was assessed for three different setups, called BC1, BC2, and BC3. Displacements were prescribed for the surface nodes on the lateral, medial and posterior side of the breast as illustrated in Table IV. BC1 and BC2 constrained 51% (54%) of the surface nodes for volunteer one (two), while BC3 constrained 24% (46%). BC3 constrained no posterior, but more lateral and anterior.

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**Table III.** Parameters of strain energy function \( \Psi \) for hyperelastic material models. The last column states mean\( (E_s/E_f) \) for strain \( \varepsilon \in [-0.25, 0] \).

<table>
<thead>
<tr>
<th>Name</th>
<th>Fatty tissue</th>
<th>Fibroglandular tissue</th>
<th>( E_s/E_f )</th>
</tr>
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<td>( \alpha_{10} )</td>
<td>( \alpha_{10} )</td>
<td>( \alpha_{10} )</td>
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<td>1.2887</td>
<td>15.0</td>
</tr>
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<td>0.1289</td>
<td>1.9330</td>
<td>20.0</td>
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<td>-7.14</td>
<td>2.07</td>
</tr>
</tbody>
</table>

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medial nodes than BC1 and BC2. The mean displacement magnitude of the posterior (medial) nodes for BC1 was 5.7 mm (1.5 mm) for volunteer one and 5.9 mm (2.3 mm) for volunteer two. BC2 assumed zero displacements of these nodes.

The volume change introduced by a boundary condition was estimated from the volume of the original and the warped surface triangulation. For BC1, the triangulation was warped by the 3D nonrigid image registration, because all displacements prescribed by BC1 were defined by the outcome of the image registration. This resulted in a volume change of 0.1% (−5.9%) for volunteer one (two) and hence we concluded that the deformation of the breast of volunteer one did not change volume. For BC2 and BC3, the triangulation was warped by the FEM model which provided the best mean displacement error for the boundary condition, resulting in an estimated volume change of −6.4% (−6.5%) for BC2 and −1.0% (−9.45%) for BC3.

4. FEM model solution

All FEM model solutions were obtained using a static or steady state analysis, i.e., the models were assumed to have relaxed to their lowest energy solution. The general approach was to use the preconditioned conjugate gradient (PCG) solver; to employ ANSYS’s automatic load step adjustment with initially one step (linear models) or four steps (nonlinear models) and maximally 500 steps; and to use an infinitesimal deformation formulation. This strategy was tested against the frontal and the sparse direct solver and a finite deformation formulation, i.e., taking account of the geometric nonlinearity.

For pure displacement approaches, nearly incompressible material can lead to predicted displacements that are much smaller than they should be, i.e., locking, or to no convergence. The mixed u-p formulation, where pressure is another solution variable, and hyperelastic models were employed to overcome these problems for the finite deformation formulation.

A continuous displacement field was derived from the node displacements by interpolation with the quadratic shape function for 10-noded tetrahedral elements.

D. Thin plate spline interpolation

The FEM solutions were compared to the result of interpolating the prescribed surface displacements by 3D thin plate spline basis functions using The Visualization Toolkit. The 3D thin plate spline basis functions are defined by $h_p(x) = |x - x_p|$, where $x_p$ denotes the 3D coordinate of the $p$th constrained node.

| TABLE IV. Configurations of three displacement boundary conditions (BC1, BC2, and BC3). The surface nodes of the FE model were classified as posterior, medial and lateral as illustrated by the black areas on the left figures. Surface nodes were either free to relax (free), displaced using the result of the 3D nonrigid registration (nreg) or fixed to remain at their initial position (fixed). BC3 contained additional nodes on the lateral and medial side towards the posterior side to compensate for the completely unconstrained posterior side. |
|---|---|---|
| BC1 and BC2 | BC3 | Configuration |
| anterior lateral posterior | anterior lateral posterior | Nodes | BC1 | BC2 | BC3 |
| lateral medial | lateral medial | Lateral | nreg | nreg | nreg |
| superior lateral inferior | otherwise | Medial | nreg | fixed | fixed |
| posterior | Otherwise | Posterior | nreg | fixed | free |

Fig. 3. Relative stress-strain relationships of fibroglandular to fatty tissue for hyperelastic models. MM$n$H denotes the approximation of MM$i$ by a neo-Hookean model and MM$MR$ represents the five-parameter Mooney-Rivlin model of MM$i$. 

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E. Landmark selection

The accuracy of each model was assessed by manually identifying for each breast 12 corresponding point landmarks in the set of images.

Reference landmarks were selected in the image of the deformed breast. Landmarks were generally placed at bifurcations, centers of small tissue regions or tips of spiculated branches, as illustrated in Figure 4. Two landmarks were usually picked in every tenth slice for volunteer one and in every seventh slice for volunteer two. Figure 5 shows the distribution of the landmarks.

The corresponding landmarks were then identified in the image showing the undeformed breast. This was repeated two more times by a single observer on different days. The average position of these three measurements was assigned to each landmark.

Figure 5 shows the displacement pattern of the landmarks for both volunteers. These differ mainly in the inferior to superior direction. This was caused by the inferior part (volunteer one) or the superior part (volunteer two) of the undeformed breast being closer to the lateral plate.

F. Error measures

The intraobserver variability of selecting a landmark was measured by the mean Euclidean distance of the three measurements to their average position. The intraobserver variability for the 24 landmarks ranged from 0.05 mm to 0.77 mm with a mean of 0.24 mm and a 90th percentile of 0.50 mm.

The performance of a model was quantified by the Euclidean distance between the position of the reference landmark and the predicted position of the corresponding landmark by
the model. This distance is called the displacement error. The distribution of the displacement errors from the 12 landmarks of each volunteer was summarized by its mean and its maximum.

Statistical significance was not assessed because of the small number of datasets. By definition the best model should be that which minimizes the displacement error measure. Any model that produced a mean (maximum) displacement error that exceeded the overall best results by less than the mean (90th percentile) intraobserver variability of 0.24 mm (0.50 mm) was also classified as best.

The sensitivity of the displacement errors was assessed with respect to four factors, namely the mean Young’s moduli ratio of fibro glandular and fatty tissue, the Poisson’s ratio, the boundary condition, and the addition of skin. The analysis was based on the model-free sensitivity index $S_i$, which measures the average reduction in variance of the displacement error for sample quantity $Y$ when fixing individual factors $F_j$. $S_i$ is given by $1-[1/N \sum_{j=1}^{N} \text{var}(Y|F_j=f_j)]/\text{var}(Y)$ where $N$ is the number of possible values for factor $F_i$ and $\text{var}(Y|F_i=f_i)$ is the variance of $Y$ after fixing factor $F_i$ to value $f_j$.

### G. Overview of tests

**Test 1 — Direct solvers:** Results obtained with the PCG solver were compared to results obtained by two direct solvers, namely the frontal and the sparse direct solver. Tests were conducted for the coarser mesh (GM1), nine models without skin (MM1–MM9), a Poisson’s ratio of 0.495 (0.2) for volunteer one (two), the accurate boundary condition (BC1) and an infinitesimal deformation formulation.

**Test 2 — Mesh density:** This test assessed whether doubling the number of mesh elements provided different results (GM1 versus GM2). The test configurations were otherwise as in Test 1 when employing the PCG solver.

**Test 3 — Finite deformation formulation:** An infinitesimal and a finite deformation formulation were compared in Test 3. MM1–MM9, a Poisson’s ratio of 0.499 and the PCG solver were employed for the infinitesimal deformation formulation. Incompressible hyperelastic models fitted to MM1–MM9, a mixed $u$-$p$ approach and the sparse direct solver were used for the finite deformation formulation. Tests were only conducted for volunteer one and BC1, since this was the only deformation that preserved volume.

**Test 4 — Linear and piecewise-linear models, skin, Poisson’s ratios, boundary conditions:** Six linear and three piecewise-linear models with and without skin (MM1–MM9, MM1S–MM9S) were compared for three boundary conditions (BC1, BC2, BC3) and Poisson’s ratios 0.499, 0.495, 0.45, 0.4, 0.3, 0.2, and 0.1. All models were solved using the coarser mesh (GM1), the PCG solver and an infinitesimal deformation formulation.

### III. RESULTS

This study assessed the influence of different tissue properties and Poisson’s ratios, boundary conditions, finite element solvers and mesh resolutions on the accuracy with which biomechanical breast models can predict the displacements of internal breast structures.

### A. Visualization of results

Figures 6(a) and 7(a) show 2D orthogonal example slices of the MR breast images before and after compressing the breast between two plates for volunteers one and two, respectively. The misalignment due to the compression is clearly visible on the difference images. Motion artifacts in the difference images were greatly reduced after 3D nonrigid registration [Figures 6(b) and 7(b)] with the mean (maximum) displacement error dropping from 6.41 mm (12.98 mm) to 0.92 mm (1.67 mm) for volunteer one and from 6.80 mm (11.85 mm) to 1.03 mm (2.74 mm) for volunteer two. Less artifact reduction was achieved by the biomechanical breast models, with the mean displacement error ranging from 1.88 mm to 4.58 mm for volunteer one [Figures 6(c)–6(h)] and from 2.12 mm to 4.01 mm for volunteer two [Figures 7(c)–7(h)].

### B. Test 1: Direct solvers

Employing the direct solvers (frontal or sparse direct) instead of an iterative solver changed the mean (maximum) displacement errors of the material models from MM1 to MM9 by less than 0.0007 mm (0.0012 mm) for both volunteers. In conclusion, employing a direct solver did not influence the accuracy of the models at the precision of the landmark selection.

### C. Test 2: Mesh density

A denser mesh (GM2) changed the mean (maximum) displacement error on average by 0.003 mm (0.004 mm) for volunteer one and by ~0.088 mm (~0.059 mm) for volunteer two. The mean (maximum) displacement error of the individual models changed by less than 0.097 mm (0.196 mm) for both volunteers. The accuracy of the models were therefore not substantially changed by a finer mesh.

### D. Test 3: Finite deformation formulation (volunteer one, BC1 only)

Finite deformation solutions were obtained for incompressible hyperelastic models using a mixed $u$-$p$ solution strategy for the volume preserving deformation, i.e., volunteer one and boundary condition BC1. These were compared to linear and piecewise-linear models that used an infinitesimal deformation formulation and a Poisson’s ratio of 0.499.

None of the finite deformation formulations improved the result of their infinitesimal counterpart. Neo-Hookean models increased, the mean (maximum) displacement errors on average by 0.07 mm (0.15 mm) when compared to the linear models and by 0.28 mm (0.34 mm) for MM7–MM9. Approximating the nonlinear material models by Mooney-Rivlin models increased the mean (maximum) displacement error by 0.14 mm (0.25 mm) on average. The volume...
changes introduced by the hyperelastic models (0.0% to 0.1%) resembled the volume change of the 3D image registration. The linear and piecewise-linear models reduced the mesh volume by 0.7%.

The mean (maximum) displacement error changed by more than 0.24 mm (0.50 mm) only when approximating MM9 by a neo-Hookean model. This worse performance is likely to have been caused by the poor approximation of the stress-strain relationship as is apparent in Figure 3. The neo-Hookean models matched the linear models very well, but still did not improve the results. It must therefore be concluded that incompressible hyperelastic models and the use of a finite deformation formulation results in a similar accuracy to linear and piecewise-linear models with a Poisson’s ratio of 0.499 and an infinitesimal deformation formulation for a volume preserving deformation.

Fig. 6. Axial, sagittal and coronal 2D example slices of volunteer one showing (a) original images with (top) compressed breast, (middle) uncompressed breast, (bottom) difference image of uncompressed and compressed breast; (b) difference image after 3D nonrigid registration; (c)–(h) difference image of the FEM deformed image and the original image of the compressed breast for results with (c), (e), (g) smallest mean displacement error and (d), (f), (h) largest mean displacement error for BC1-3. The mean displacement error DE are listed for each case.

Fig. 7. Axial, sagittal and coronal 2D example slices of volunteer two of the original image with (top) compressed breast, (middle) uncompressed breast, (bottom) difference image of uncompressed and compressed breast; (b) difference image after 3D nonrigid registration; (c)–(h) difference image of the FEM deformed image and the original image of the compressed breast for results with (c), (e), (g) smallest mean displacement error and (d), (f), (h) largest mean displacement error for BC1-3. The mean displacement error DE are listed for each case.
E. Test 4: Linear and piecewise-linear models, skin, Poisson’s ratios, boundary conditions

The effects of the average elastic ratio of fibroglandular and fatty tissue, and the Poisson’s ratio on the displacement error are illustrated for BC1 and models without skin in Figures 8 and 9. Table V provides an overview of the main results:

(i) For volunteer one and boundary condition BC1, the best results were achieved by material models where fibroglandular tissue was at most 4 times stiffer than fatty tissue and by MM3S while keeping Poisson’s ratios high ($\nu \in \{0.499, 0.495\}$), see Figure 8. These models reduced the mean (maximum) displacement error to 1.96 mm (3.43 mm). Assuming that no motion had occurred at the posterior and medial side BC2 caused substantially higher errors, see Table V. Best results for BC2 required low Poisson’s ratios ($\nu \in \{0.1, 0.2, 0.3\}$), due to the introduction of volume changes. Keeping the posterior side unconstrained (BC3) was better than assuming no motion at the posterior side (BC2) but was worse than BC1. Best models for BC3 had low Young’s moduli ratios and high Poisson’s ratios. Very similar contour plots were achieved when adding skin with a Young’s modulus of 10 kPa, with the mean (maximum) displacement error of any model changing by less than 0.21 mm (0.36 mm). The mean and maximum displacement error was most sensitive to the boundary condition, see Table VI. The most important factor after fixing the boundary condition was either the Poisson’s ratio for BC1 apart from the mean error of MM1–MM6, for BC2 and for the maximum error of MM7–MM9 for BC3 or the Young’s moduli ratio. Thin plate spline interpolation of the prescribed surface displacements provided only for BC1 a mean and a maximum displacement error that was similar to the best FE models, see Table V.

(ii) For volunteer two, the nonrigid registration introduced a volume change of −5.9%. Substantially worse results were therefore obtained at high Poisson’s ratios, see Figure 9 and Table V. Best results for BC1 were achieved by all models but MM6, MM9, MM9S while keeping Poisson’s ratios low ($\nu \in \{0.1, 0.2, 0.3\}$). Assuming no motion at the posterior and medial side (BC2) introduced lower errors than for volunteer one (Table V). Not constraining the posterior side (BC3) provided similar results as BC2. Best results for BC3 were achieved by linear models with skin and a low Poisson’s ratio. Adding skin had a bigger impact on the results than for volunteer one, with the mean (maximum) displacement error changing on average, by −0.09 mm (−0.42 mm) and up to 0.31 mm (2.21 mm). The mean and maximum displacement error was most
sensitive to the Poisson’s ratio, see Table VI. The most important factor after fixing the Poisson’s ratio was the boundary condition apart from one case (mean error of MM7–MM9 for \( v = 0.1 \)) where the results were more sensitive to the Young’s moduli ratio. Thin plate spline interpolation of the boundary constraints (Table V) provided worse results than the best FE models.

In conclusion, nine material models achieved for BC1 and appropriate Poisson’s ratios best results for both volunteers, namely all material models where fibroglandular tissue was at most 4 times stiffer than fatty tissue and MM3S. Their mean (maximum) displacement error was on average 2.11 mm (3.37 mm). Less accurate or less constrained boundary conditions (BC2, BC3) led to higher errors. Keeping the posterior side unconstrained (BC3) was better for volunteer one than assuming no motion at this side (BC2). The boundary condition and the Poisson’s ratio had the highest influence on the displacement error.

**IV. CONCLUSIONS**

This study evaluated the accuracy with which biomechanical breast models were able to predict the in-vivo displacements of internal breast structures for a pre- and a post-menopausal volunteer. The first objective of this study was to find a suitable FEM configuration that can produce plausible deformations during DCE MR mammography acquisition. For this purpose, any model which achieved, for accurate boundary conditions, a mean (maximum) displacement error that exceeded the best result by less than 0.24 mm (0.50 mm) for both volunteers was regarded as suitable. The second aim was to investigate what modelling aspects are most important.

Biomechanical breast models were able to predict the displacements of internal breast structures (i.e., 12 corresponding anatomical landmarks) for accurate boundary conditions (BC1), appropriate Poisson’s ratios and suitable elastic properties (fibroglandular tissue is at most 4 times stiffer than fatty tissue) to a mean accuracy of 2.0 mm (volunteer one) and 2.2 mm (volunteer two) for a deformation that introduced a mean displacement of 6.4 mm and 6.8 mm, respectively. Maximum errors reduced from 13 mm (volunteer one) and 11.8 mm (volunteer two) to 3.4 mm and 3.3 mm, respectively. These suitable elastic properties are within the range of values reported by in-vivo elastography studies.24–27

The models were generally more sensitive to the choice of Poisson’s ratio or boundary condition than to the elasticity properties. Lower Poisson’s ratios improved the prediction when volume changes needed to be modelled. Assuming no motion at the posterior and medial nodes, as often done when modelling breast compressions,1–7 led to worse results for volunteer one than keeping the posterior nodes unconstrained.

Employing direct solvers or a finer mesh of improved quality did not influence the accuracy. Using hyperelastic material models and a finite deformation formulation did not improve the accuracy.

The robustness of the results to the elastic properties were probably caused by modelling only a few tissue types and by...
the FEMs being greatly constrained by surface displacements and hence acting as interpolants. A similar robustness was reported by Ruiter et al.6,7 The results of this study compare favorably with the performance reported by Azar et al.1 and Ruiter et al.6,7 most likely because of improved boundary conditions and much finer meshes. The 3D image registration achieved mean errors of about 1.0 mm using just the information provided by the images and a mechanical unconstrained transformation model. Whether such an accuracy could be obtained prospectively with a patient specific, heterogeneous biomechanical model and matching boundary conditions, remains to be seen.

Predicting breast deformations with the reported accuracy would improve the surgical precision for nonpalpable lesions, support the registration of x-ray and DCE MR mammograms and provide plausible breast deformations for validation and teaching purposes. The main challenge in these applications is, therefore, to establish accurate boundary conditions. When registering, for example, a preoperative MR mammogram to the patient for surgical planning, this could require image-visible surface markers, surface extraction of the patient’s breast using a stereo camera system, surface registration and ultrasound based registration of the posterior side and we are developing a system to do just that.

The applied forces in these experiments have not been measured and hence it is not known whether these would justify the 6% volume change observed for volunteer two on the basis that venous blood pressure values were exceeded. During DCE MR mammography acquisition, however, no significant change in external force is applied.

Our investigation of the most important modelling aspects is limited because only two volunteers were assessed, only one deformation scenario was evaluated and the applied compressions were much lower than during x-ray mammography. Nevertheless, we have no evidence to suggest that the properties of the breast tissue examined are anything but typical and the results of this study provide already valuable information for the practical clinical situation where the boundaries of the breast are known rather accurately.

In conclusion, first all material models where fibroglandular tissue was no greater than 4 times stiffer than fatty tissue provided the best results for both volunteers and can be recommended in conjunction with a high Poisson’s ratio for the simulation of plausible breast deformations during DCE MR mammography. Second, the accuracy of the model was more affected by the choice of boundary condition or Poisson’s ratio than by the choice of elastic material proper-

<table>
<thead>
<tr>
<th>Smallest error (mm)</th>
<th>Best FE models</th>
<th>Error (mm)</th>
<th>TPS interp.Error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Max</td>
<td>Material model</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>Volunteer one</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BC1</td>
<td>1.88</td>
<td>3.33</td>
<td>mean($E_g/E_f$) &lt; 5, MM3S</td>
</tr>
<tr>
<td>BC2</td>
<td>2.75</td>
<td>7.12</td>
<td>mean($E_g/E_f$) &lt; 4, MM9(S)</td>
</tr>
<tr>
<td>BC3</td>
<td>2.25</td>
<td>4.76</td>
<td>mean($E_g/E_f$) &lt; 4, MM8S</td>
</tr>
<tr>
<td>Volunteer two</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BC1</td>
<td>2.12</td>
<td>3.15</td>
<td>all but MM6, MM9(S)</td>
</tr>
<tr>
<td>BC2</td>
<td>2.36</td>
<td>3.85</td>
<td>all but MM5, MM6</td>
</tr>
<tr>
<td>BC3</td>
<td>2.52</td>
<td>3.80</td>
<td>MM1S-MM6S</td>
</tr>
</tbody>
</table>

Table VI. Sensitivity analysis of the displacement error. The sensitivity index $S_i$ measures the average variance reduction of the mean or maximum displacement error when fixing the listed factor. Influential factors are indicated by large $S_i$ values.

<table>
<thead>
<tr>
<th>Fixed factor</th>
<th>Volunteer one MM1–MM6</th>
<th>MM7–MM9</th>
<th>Volunteer two MM1–MM6</th>
<th>MM7–MM9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s moduli ratio $E_g/E_f$</td>
<td>Mean</td>
<td>Max</td>
<td>Mean</td>
<td>Max</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu$</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Boundary condition</td>
<td>52</td>
<td>92</td>
<td>59</td>
<td>93</td>
</tr>
<tr>
<td>Skin</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

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ties. Further studies are required to reconfirm the importance of these modelling factors for other applications.

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32Direct solvers use a direct elimination process to solve a system of linear equations. Iterative solvers (e.g., PCG solver) start from an initial estimate of the solution and then improve it until convergence. Direct solvers are generally more robust, but require more memory and are slower.