Assessing the effect of electron density in photon dose calculations

J. Seco
Francis H. Burr Proton Therapy Center, Massachusetts Hospital, Harvard Medical School, 30 Fruit Street, Boston, Massachusetts 02114

P. M. Evans
Joint Department of Physics, Institute of Cancer Research and The Royal Marsden NHS Foundation Trust, Downs Road, Sutton, Surrey SM2 5PT, United Kingdom

(Received 6 July 2005; revised 28 November 2005; accepted for publication 29 November 2005; published 31 January 2006)

Photon dose calculation algorithms (such as the pencil beam and collapsed cone, CC) model the attenuation of a primary photon beam in media other than water, by using pathlength scaling based on the relative mass density of the media to water. In this study, we assess if differences in the electron density between the water and media, with different atomic composition, can influence the accuracy of conventional photon dose calculations algorithms. A comparison is performed between an electron-density scaling method and the standard mass-density scaling method for (i) tissues present in the human body (such as bone, muscle, etc.), and for (ii) water-equivalent plastics, used in radiotherapy dosimetry and quality assurance. We demonstrate that the important material property that should be taken into account by photon dose algorithms is the electron density, and not the mass density. The mass-density scaling method is shown to overestimate, relative to electron-density predictions, the primary photon fluence for tissues in the human body and water-equivalent plastics, where 6%–7% and 10% differences were observed respectively for bone and air. However, in the case of patients, differences are expected to be smaller due to the large complexity of a treatment plan and of the patient anatomy and atomic composition and of the smaller thickness of bone/air that incident photon beams of a treatment plan may have to traverse. Differences have also been observed for conventional dose algorithms, such as CC, where an overestimate of the lung dose occurs, when irradiating lung tumors. The incorrect lung dose can be attributed to the incorrect modeling of the photon beam attenuation through the rib cage (thickness of 2–3 cm in bone upstream of the lung tumor) and through the lung and the oversimplified modeling of electron transport in convolution algorithms. In the present study, the overestimation of the primary photon fluence, using the mass-density scaling method, was shown to be a consequence of the differences in the hydrogen content between the various media studied and water. On the other hand, the electron-density scaling method was shown to predict primary photon fluence in media other than water to within 1%–2% for all the materials studied and for energies up to 5 MeV. For energies above 5 MeV, the accuracy of the electron-density scaling method was shown to depend on the photon energy, where for materials with a high content of calcium (such as bone, cortical bone) or for primary photon energies above 10 MeV, the pair-production process could no longer be neglected. The electron-density scaling method was extended to account for pair-production attenuation of the primary photons. Therefore the scaling of the dose distributions in media other than water became dependent on the photon energy. The extended electron-scaling method was shown to estimate the photon range to within 1% for all materials studied and for energies from 100 keV to 20 MeV, allowing it to be used to scale dose distributions to media other than water and generated by clinical radiotherapy photon beams with accelerator energies from 4 to 20 MV. © 2006 American Association of Physicists in Medicine. [DOI: 10.1118/1.2161407]

Key words: photon dose calculation algorithm, mass-density heterogeneity correction, electron-density heterogeneity correction

I. INTRODUCTION

Photon beam dose algorithms, used in radiotherapy treatment planning, are based on Monte Carlo calculations of primary photon beam attenuation and dose-deposition kernels in water.1 When applying these dose algorithms to media other than water, a heterogeneity correction is used. Heterogeneity corrections performed to nonwater materials have evolved with computing speed and the understanding and modeling of the radiation transport and interaction through regions of varying density. Algorithms such as a pencil beam (PB), a collapsed cone (CC), Batho, etc., all have a heterogeneity pathlength correction based on the mass-density value (\(\rho\)) per voxel obtained from CT data, which is known as an equivalent pathlength (EPL).1 The EPL method scales the beam dose distribution to take into account changes with depth of the primary fluence in a medium other than water.
O’Connor² and Sontag and Cunningham³ suggested the basis of the EPL method, which relates the dose in two media of different density but equal atomic composition, irradiated by the same external photon beam. O’Connor⁷ proposed that the ratio of scattered photon fluence to that of primary fluence at any point in two media are equal provided all geometric distances (including field sizes) were inversely scaled by the mass density. The O’Connor theorem relates the dose in two media of different density and equal atomic composition, both irradiated by the same external beam.

Monte Carlo has been used to assess how conventional dose calculation algorithms such as PB and CC perform in or around heterogeneities,¹⁺⁻⁸ where heterogeneity corrections use the EPL method. Ahnesjö⁹ showed that the CC algorithm overestimated the dose in the lung for small fields due to an overestimation of the primary dose and oversimplified modeling of electron transport in convolution algorithms. In regions of electronic disequilibrium, the CC was also shown to differ from MC predictions. These differences occurred at interfaces between heterogeneities and for small field sizes, where CC again overestimated the dose from primary photons. Although the CC algorithm calculates a dose reduction in the region of lateral electronic disequilibrium, it does not calculate the dose from primary photons accurately. Woo and Cunningham⁵ also observed this when evaluating CC in low-density media like cork. They showed that CC overestimated the dose at the proximal distal interfaces and within lung tissue and that the differences increased with increasing energy.

The purpose of this paper is to (a) assess if differences in the electron density between water and other media, with a different atomic composition, can influence the accuracy of conventional photon dose calculations algorithms; (b) determine a universal heterogeneity correction, based on electron density that can be applied to conventional photon dose algorithms to scale the dose distributions correctly for media other than water; (c) compare this new electron-density heterogeneity correction with the standard EPL mass-density heterogeneity correction in (i) tissues present in the human body (such as bone, muscle, etc.) and in (ii) water-equivalent plastic materials, used in radiotherapy quality assurance phantoms; and (d) assess the accuracy of this electron-density heterogeneity correction as a function of the photon energy.

II. BACKGROUND CONVERTING CT NUMBERS TO MASS DENSITY FOR RADIOTherapy DOSE CALCULATION

Computerized tomography (CT) measures the spatial distribution of the linear attenuation coefficient $\mu(x,y)$.¹⁰⁻¹² However, the physical quantity $\mu(x,y)$ is strongly dependent on the spectral energy used during the CT scan. Therefore, the attenuation coefficient is displayed as a CT number relative to the attenuation of water and defined by the following expression:

$$ N_{\text{CT}} = \frac{\mu_{\text{mat}} - \mu_{\text{H}2\text{O}}}{\mu_{\text{H}2\text{O}}} \times 1000, $$

where $\mu_{\text{mat}}(x,y)$ and $\mu_{\text{H}2\text{O}}(x,y)$ are the linear attenuation coefficients for a material mat and water, respectively, and the CT number, $N_{\text{CT}}$, comes in units of Hounsfield Units (HU), with each HU equivalent to 0.1% of the linear attenuation of water.¹² The $N_{\text{CT}}$ definition given in Eq. (1) is such that water with a density of 1.0 g/cm³ has a CT number of 0. Since Hounsfield numbers are proportional to x-ray attenuation, they depend primarily on the electron density and, especially for heavy atoms, on the atomic number, because of the dominant interaction at the CT scanning energies in Compton scattering. However, the electron density is a quantity that is not used in commercial planning systems like Helax-TMS,¹³ Pinnacle,¹⁴ etc. The derivation of interaction parameters needed by the dose engines is instead based on a mapping from Hounsfield numbers to mass density.

For every CT scanner used to obtain patient images for radiotherapy treatment planning, a CT calibration phantom with inserts with known mass densities is usually scanned to obtain the CT numbers associated with those mass densities.¹³,¹⁴ In Fig. 1, we present an example CT calibration curve to mass density that was obtained for materials present in the human body such as, air, lung, water, muscle, and bone. In addition, the CT calibration curve to electron density is also presented for a comparison. Both curves were normalized respectively to the mass density and electron density of water. During planning, the treatment planning system (TPS) reads in the CT numbers and then determines the mass density of each voxel of the patient using the selected CT to the mass-density calibration curve (see Fig. 1), while the electron-density curve is not used in the commercial dose calculation algorithms. The mass density is used to look up mass attenuation coefficients and is used for mass-density EPL scaling during superposition, based on the O’Connor theorem.¹² The mass density is also used to scale the dose deposition kernel during the superposition to account for the effects of heterogeneities on scattered radiation.¹¹² This is accomplished by tracing a ray line between a TERMA (total energy released per unit mass) interaction site and the dose computation point and accumulating the radiological EPL distance along the ray. In the present work, we assess if differences observed in Fig. 1 between mass density and electron density can influence the accuracy of conventional photon dose calculations algorithms.

III. METHODS AND MATERIALS

A. Mass-density heterogeneity correction used in commercial photon beam dose algorithms

The idea of the EPL heterogeneity correction methods is to scale the dose distribution, in a medium other than water and to take into account changes with depth in the primary fluence in this media relative to water. The heterogeneity corrections are usually obtained using a ray-tracing procedure, where increments along a ray line from the source ac-
count for mass-density variations along its trajectory. The increments are usually found using mass densities. The EPL is given by the following expression:

$$EPL = \sum_i l_i \frac{\rho_i}{\rho_{H_2O}},$$

where $l_i$ is the physical ray line length through medium $i$, $\rho_i$ is the mass density of $i$, and $\rho_{H_2O}$ is the mass density of water. The EPL heterogeneity correction method is based on the O'Connor theorem, which states that the ratio of the secondary scattered photon fluence to that of primary photon fluence is constant in two media, provided all geometric distances are scaled inversely with mass density.

The photon dose algorithms in many commercial treatment planning systems are based on convolution models calculated in water and adapted to media other than water using EPL scaling methods. The energy deposition by secondary particles around a primary photon interaction site is, in homogeneous media, independent of the location of the site and suitable for description by a precalculated distribution. Full three-dimensional (3-D) superposition/convolution methods such as PB and CC are based, respectively, on pencil and point kernels.

For the PB case, the dose distribution in water is generated for monoenergetic photon beams using Monte Carlo and then scaled using the EPL method to account for heterogeneities only in the direction of the incident photon fluence. For the CC case, the energy deposition kernel is obtained using a Monte Carlo simulation of photons forced to interact at a certain point in water. The dose calculation is a two-step procedure. First, the total energy released in the material $T(r)$ (also known as TERMA), is calculated for the applied radiotherapy field, where heterogeneities are accounted for in the direction of the incident photon fluence using the EPL method. Then, this is convolved with the energy deposition kernels, which are also scaled to take into account heterogeneities. Ahnesjö and Aspradakis describe the PB and CC dose algorithms in detail.

### B. Electron-density heterogeneity correction for photon beam dose algorithms

In the present study, we define the electron-density heterogeneity correction, eEPL, for photon beams by the following expression:

$$eEPL = \sum_i l_{ei} \frac{\rho_{ei}}{\rho_{H_{2O}}},$$

where $l_{ei}$ is the physical ray line length through medium $i$, $\rho_{ei}$ is the electron density of $i$, and $\rho_{H_{2O}}$ is the electron density of water. The electron density of a medium med, $\rho_{med}$, is given by

**Fig. 1.** Example CT calibration curves are presented, which allow the conversion of CT numbers (in Hounsfield units, HU) to either mass density or electron density. Both curves are normalized, respectively, to the mass density and electron density of water.
cal atomic composition but different mass densities, the pro-
the case of the O’Connor theorem for media with an identi-
cient, any media is characterised by the linear attenuation coeffi-
Compton scattering, etc. Primary photon beam attenuation in
The scattered photon fluence appears as a consequence of the
interacting, which is defined by
where the linear attenuation coefficient is related to the av-
med to equal that of water, the linear attenuation coeffi-
commercial to that of water, \( \mu_{H_2O} \). In
in the case of the O’Connor theorem for media with an identi-
composition but different mass densities, the propor-
constantly is the ratio of mass densities:
\[ \mu_{med} = \frac{\rho_{med}}{\rho_{H_2O}} \Rightarrow EPL(l_{H_2O}) = \frac{\rho_{med}}{\rho_{H_2O}} l_{med}, \]
where the linear attenuation coefficient is related to the av-
average length the photon will travel in a media med before
interacting, which is defined by \( l_{med} = 1/\mu_{med} \) and \( l_{H_2O} = 1/\mu_{H_2O} \) for water. The linear attenuation coefficient of
med is defined by the sum of the various photon interaction processes that may occur at any set energy
for radiotherapy photon beams,
\[ \mu_{med}^{PE} = \mu_{med}^{Rayleigh} + \mu_{med}^{Compton} + \mu_{med}^{PP}, \]
where PE and PP are, respectively photo-electric and pair-production.
The electron-density heterogeneity correction proposed in
Eq. (3) is obtained by assuming that the primary photon beam is attenuated in med only by Compton interactions.
This is a very good approximation for radiotherapy beams with photon energies between 100 keV and 10 MeV, where
the Compton interaction is the dominant interaction process. The Compton linear attenuation coefficient, \( \mu^{Compton}_{med} \), for a
generic medium med is
\[ \mu^{Compton}_{med} = \frac{\langle Z/A \rangle_{med} \rho_{med} \times \sigma_{KN}}{\rho_{med} \times \sigma_{KN}}, \]
where \( \langle Z/A \rangle_{med} \rho_{med} \) is the electron density of med and \( \sigma_{KN} \) is the
Klein–Nishina cross section for an interaction between a
photon and a single static and free electron. Equation (3) is
then obtained by dividing the linear attenuation coefficient of
med by that of water:
\[ \frac{\mu_{med}}{\mu_{H_2O}} = \frac{\mu^{Compton}_{med}}{\mu^{Compton}_{H_2O}} \Rightarrow l_{med} = \frac{l_{H_2O}}{\mu_{H_2O} \rho_{med}}, \]
where \( l_{med} = 1/\mu_{med} \) and \( l_{H_2O} = 1/\mu_{H_2O} \). Note that because
\( Z/A \) is approximately \( 1/2 \) for most chemical elements in the
body (except hydrogen), the Compton linear attenuation coefficient of a compound will depend significantly on the hy-
drogen content of the material.
A comparison has been performed between the mass-
density EPL and the electron-density eEPL correction meth-
ods for human tissues and water-equivalent plastic substitutes, which have very different electron-density values and
that are used in radiotherapy.

C. Mass-density and electron-density characteristics of human tissues

A set of tissues taken from ICRP 44,15 ICRP 23,16 and the
HELAX-TMS planning system (Nucletron B.V., Veenendaal, The Netherlands) was studied. The tissues and their atomic
composition are given in Table I. The values presented in
Table I are in fraction by weight of each elemental compo-
tent. The tissues indicated in Table I are those usually used
in radiotherapy dosimetry with Monte Carlo, PB, or CC dose
calculations. The mass density and \( Z/A \) values are pre-
sented at the bottom of the table, where the electron density
can be obtained for each tissue using Eq. (3).

D. Water-equivalent plastics used in dosimetry and quality assurance

It is common practice to use water-equivalent plastic
phantoms for basic quality assurance of high-energy photon beams. A plastic phantom is considered water equivalent if it
has the same absorption and scattering characteristics for
photons and electrons as water. However, since radiation in-
teractions depend on the density and atomic composition of
the material, as well as the energy of the photons and elec-
trons, it is impossible for the two materials to be strictly
equivalent over the whole spectrum of energies used in ra-
diation therapy.

Many materials have been investigated as substitutes for
water.17–21 We have compared the mass-density and electron-
density heterogeneity correction methods in several water-
equivalent plastics, given in Table II. These water-equivalent
plastics are used in many clinics for quality assurance and
calibration at depths of either 5 or 10 cm. The water-
equivalent plastics have approximately the same mass den-
sity but very different electron density (cf. Table II).

E. Homogeneous multilayer phantom used to evaluate photon beam attenuation in a media

The FLURZnrc Monte Carlo user code was used to model
a homogeneous cylindrical phantom, in order to simulate the
photon beam attenuation in a homogenous medium made of

Medical Physics, Vol. 33, No. 2, February 2006
the materials presented in Tables I and II. FLURZnrc calculated the photon fluence in the cylindrical RZ geometry and was used to assess which heterogeneity correction method: (a) EPL or (b) eEPL correctly predicts the scattered photon fluence as the photon beam is attenuated in a media other than water.

The RZ phantom was made up of 60 slabs with a water-equivalent thickness of 0.5 cm and two concentric cylinders of radius 0.01 and 100 cm, to model the attenuation of a pencil beam. Therefore, in media other than water, the thickness of each slab was calculated using either (a) the standard mass density EPL method defined in Eq. (2), where EPL =0.5 cm, or (b) the electron-density eEPL method defined in Eq. (3), where eEPL=0.5 cm. For example, in the case of bone, the mass density of bone relative to water is $\rho_{\text{Bone}}/\rho_{\text{H}_2\text{O}} = 1.85$, while the electron density relative to water is approximately $e_{\text{Bone}}/e_{\text{H}_2\text{O}} = 1.72$. Thus, for the standard mass-density EPL method the slab thickness is approximately $l_{\text{med}} = 0.27\text{ cm}$, while for the electron-density eEPL method the slab thickness is $l_{\text{med}} = 0.29\text{ cm}$. A 1 MeV photon pencil beam (of width 0.01 cm) was used to irradiate the

### Table I. Atomic composition of various tissues in the human body (Refs. 15 and 16), ordered from lowest mass-density value, $\rho$, to the highest value. The effective atomic number to atomic weight ratio of the compound is \(Z/A = \sum_i(Z_i/A_i)f_i\), where \((Z/A)\) is the atomic number to atomic weight ratio of chemical element \(i\) and \((f_i)\) is the fraction of mass of the element in the compound presented in the table. Chemical elements presented in the table are identified by their chemical symbol.

<table>
<thead>
<tr>
<th>Z</th>
<th>Z/A</th>
<th>Water (H(_2)O)</th>
<th>Air</th>
<th>Lung</th>
<th>Adipose</th>
<th>Tissue</th>
<th>Muscle</th>
<th>Cartilage</th>
<th>Bone</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.992</td>
<td>11.19</td>
<td>10.30</td>
<td>11.40</td>
<td>10.12</td>
<td>10.20</td>
<td>9.60</td>
<td>3.40</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.500</td>
<td>59.80</td>
<td>59.80</td>
<td>59.80</td>
<td>59.80</td>
<td>59.80</td>
<td>59.80</td>
<td>59.80</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>0.500</td>
<td>7.10</td>
<td>7.10</td>
<td>7.10</td>
<td>7.10</td>
<td>7.10</td>
<td>7.10</td>
<td>7.10</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>0.500</td>
<td>27.80</td>
<td>27.80</td>
<td>27.80</td>
<td>27.80</td>
<td>27.80</td>
<td>27.80</td>
<td>27.80</td>
<td></td>
</tr>
<tr>
<td>Na</td>
<td>0.478</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>Mg</td>
<td>0.494</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>0.484</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>0.499</td>
<td>2.60</td>
<td>2.60</td>
<td>2.60</td>
<td>2.60</td>
<td>2.60</td>
<td>2.60</td>
<td>2.60</td>
<td></td>
</tr>
<tr>
<td>Cl</td>
<td>0.479</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>Ar</td>
<td>0.451</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>0.486</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>Ca</td>
<td>0.499</td>
<td>2.20</td>
<td>2.20</td>
<td>2.20</td>
<td>2.20</td>
<td>2.20</td>
<td>2.20</td>
<td>2.20</td>
<td></td>
</tr>
</tbody>
</table>

\(\rho (\text{g/cm}^3)\): 1.000, 1.020, 1.035, 1.045, 1.060, 1.127, 1.190

\(\langle Z/A \rangle\): 0.555, 0.540, 0.545, 0.540, 0.539, 0.536, 0.538, 0.547, 0.539

### Table II. Atomic composition of various water-equivalent materials (Refs. 17–21), which are used for radiotherapy phantoms, ordered from lowest mass-density value, $\rho$, to the highest value. The effective atomic number to atomic weight ratio of the compound is \(Z/A = \sum_i(Z_i/A_i)f_i\), where \((Z/A)\) is the atomic number to atomic weight ratio of chemical element \(i\) and \((f_i)\) is the fraction of mass of the element in the compound presented in the table. Chemical elements presented in the table are identified by their chemical symbol.

<table>
<thead>
<tr>
<th>Z</th>
<th>Z/A</th>
<th>Water ((\text{H}_2\text{O}))</th>
<th>Plastic (\text{H}_2\text{O})</th>
<th>Polystyrene</th>
<th>A-150</th>
<th>PMMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.992</td>
<td>11.19</td>
<td>8.21</td>
<td>0.80</td>
<td>1.40</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.500</td>
<td>66.33</td>
<td>62.72</td>
<td>5.23</td>
<td>1.84</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>0.500</td>
<td>2.21</td>
<td>2.40</td>
<td>3.51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>0.500</td>
<td>88.81</td>
<td>17.94</td>
<td>5.32</td>
<td>31.96</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0.474</td>
<td>20.65</td>
<td>19.90</td>
<td>3.14</td>
<td>3.16</td>
<td></td>
</tr>
<tr>
<td>Mg</td>
<td>0.494</td>
<td>2.20</td>
<td>2.30</td>
<td>1.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cl</td>
<td>0.479</td>
<td>0.40</td>
<td>0.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ca</td>
<td>0.499</td>
<td>2.20</td>
<td>2.30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti</td>
<td>0.459</td>
<td>2.20</td>
<td>1.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Br</td>
<td>0.438</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(\rho (\text{g/cm}^3)\): 1.000, 1.020, 1.035, 1.045, 1.060, 1.127, 1.190

\(\langle Z/A \rangle\): 0.555, 0.540, 0.545, 0.540, 0.539, 0.536, 0.538, 0.547, 0.539
cylindrical phantom. This energy was selected because it is indicative of the average energy of a 6 MV treatment beam.

IV. RESULTS

A. Mass-density versus electron-density heterogeneity correction for a monoenergetic photon beam

The results were analyzed by plotting the ratio of the primary photon fluence in a material to that in water, corrected for differences in either the mass density or the electron density of the material relative to water. This ratio indicates how good the EPL or the eEPL methods are at estimating the fluence in a medium, if the fluence in water is known. If the heterogeneity correction is a good estimate of the photon attenuation in a medium other than water, then this ratio of primary photon fluence in a material to that of water should be 1.00.

1. A 1 MeV photon beam irradiating human body tissues

In Figs. 2(a) and 2(b), the ratio of primary photon fluence in human tissues to fluence in water for a 1 MeV beam is presented using, respectively, the mass density EPL and the electron-density eEPL estimates of the water-equivalent depth for tissues in the human body.

While the electron-density eEPL scaling method correctly predicts the primary photon fluence of the human tissues relative to water.

In the case of bone and air, the EPL method predicts significantly higher primary photon fluence than in water because their atomic compositions are very different from that of water. Both air and bone have a lower hydrogen content than water; therefore their effective atomic number to atomic weight ratio is very different from that of water, where \( \langle Z/A \rangle_{\text{bone}}/\langle Z/A \rangle_{\text{H}_2\text{O}} = 0.928 \) and \( \langle Z/A \rangle_{\text{air}}/\langle Z/A \rangle_{\text{H}_2\text{O}} = 0.899 \). For the remaining materials presented in Fig. 2, i.e., lung, adipose, tissue, muscle, and cartilage, the hydrogen content is of the order of that in water (cf. Table I). Therefore, smaller differences are observed for the primary photon fluence prediction in these materials relative to that of water. In the case of the eEPL scaling method, since it takes into account the electron density of the media, it therefore accounts for differences in the hydrogen content between the tissues in the human body and water. Thus eEPL accurately predicts the primary photon beam attenuation in media like bone and air, which have a significantly different atomic composition than water. The eEPL scaling method predicts to within 0.5\% the photon fluence in tissues of the human body relative to water, for depths in water up to 20 cm.

2. A 1 MeV photon beam irradiating water-equivalent plastics

The primary photon fluence in water-equivalent plastics is presented in Figs. 3(a) and 3(b), using, respectively, the EPL and the eEPL estimates of the water-equivalent slab thickness for that material. It is possible to observe that at the depths of 5 and 10 cm in water, the EPL method predicts a...
systematically higher fluence of 1%–2%, in the medium relative to water, while the eEPL accurately predicts the medium fluence relative to water at the same depths.

This difference is largest for the plastics: RW3, polystyrene, and PMMA, which have a lower content of hydrogen than water. This lower content of hydrogen leads to a lower effective atomic number to atomic weight ratio to that of water, where the values are $\frac{Z}{\omega}/H_2O = 0.966$, $\frac{Z}{\omega}/\text{Polystyrene}/H_2O = 0.969$, and $\frac{Z}{\omega}/\text{PMMA}/H_2O = 0.971$. Since the eEPL scaling method correctly accounts for differences in the electron density of the water-equivalent plastics relative to water; it therefore accurately predicts the primary photon fluence in these water-equivalent materials. The eEPL scaling method predicts to within 0.5% the photon fluence in water-equivalent plastics relative to water, for depths in water up to 20 cm.

B. Accuracy of mass-density and electron-density heterogeneity corrections with photon energy

The accuracy of the mass-density and the electron-density heterogeneity corrections will depend on the photon energy at which these energies are performed. We evaluate for (a) tissues in the human body and (b) water-equivalent plastics the accuracy of the mass-density and the electron-density heterogeneity corrections as a function of the photon energy. The results were analyzed by plotting the ratio of the photon attenuation lengths in water relative to a material, $\frac{\rho_{\text{med}}(H_2O/l_{\text{med}})}{\rho_{\text{med}}(H_2O/l_{\text{med}})}$ for tissues defined in Table I, is presented, where $\rho_{\text{med}}$ is either the ratio of the mass density or the electron density of water relative to the medium, respectively, for the EPL and the eEPL methods. The photon attenuation lengths $l_{\text{H}_2\text{O}}$ and $l_{\text{med}}$ were calculated using attenuation coefficients from the XCOM photon cross-section database, where the values were divided, respectively, by the mass density and electron density of water and med in order to allow a comparison with the EPL and eEPL models.

For energies between 100 keV and 7 MeV, the EPL model tends to systematically predict larger photon attenuation lengths in water for the tissues defined in Table I. For lung, tissue, muscle, and cartilage the EPL method predicts a photon attenuation length that is, on average, 2% larger than the $\frac{\rho_{\text{med}}(H_2O/l_{\text{med}})}{\rho_{\text{med}}(H_2O/l_{\text{med}})}$ value. In the case of bone and air, a larger discrepancy is observed between the EPL estimate and the $\frac{\rho_{\text{med}}(H_2O/l_{\text{med}})}{\rho_{\text{med}}(H_2O/l_{\text{med}})}$ value (maximum differences are, respectively, 7% and 10% and occurring for photon energies close to 1 MeV), with the EPL model providing a poor estimate. The differences observed between $\frac{\rho_{\text{med}}(H_2O/l_{\text{med}})}{\rho_{\text{med}}(H_2O/l_{\text{med}})}$ and the EPL estimate of the photon attenuation length for all materials in Table I are a consequence of the lower content of hydrogen in these media relative to water. In the case of bone and air, where the discrepancies are larger, the hydrogen content is significantly lower than in water.
For the same energy range (100 keV to 7 MeV), the eEPL model provides a significantly better estimate of the photon attenuation length in water than EPL, for nearly all materials defined in Table I, except bone. For this energy interval, the eEPL estimate is within 2% of \( l_{\text{med}} / l_{\text{H}_2\text{O}} \) for all materials in Table I, except bone. In the case of bone, the eEPL method provides a good estimate of the photon attenuation length to within 3%, for photon energies between 100 keV and 4.0 MeV. However, for photon energies above 4.0 MeV, the eEPL method incorrectly estimates the photon attenuation length. The increase with energy of \( l_{\text{med}} / l_{\text{H}_2\text{O}} \) is a consequence of the increase in the probability of pair production occurring in bone, which is not accounted for in the eEPL model. The high probability of pair production in bone is a consequence of the high content of calcium (Ca) and phosphor (P), which have a significantly higher Z value than chemical elements present in other media in Table I and because the probability of pair production increases with \( Z^2 \).

The photon fluence spectrum of a \( 10 \times 10 \text{ cm}^2 \) field is presented in Fig. 4(c) for a clinical 6 MV photon beam, obtained at 100 cm source to surface distance.
rection on a clinical 6 MV photon beam for tissues in the human body. The energy spectrum was normalized to unity such that it could be used to perform weighted calculations of the percentage difference between the EPL or the eEPL method and the photon attenuation length \( l_{\text{H}_2\text{O}} \) in water. The results obtained are presented in Table III, where the values represent how accurately primary beam attenuation is modeled by either the EPL or the eEPL methods for the 10 \( \times 10 \) cm\(^2\) energy spectrum. The eEPL model accurately predicts, to within 1%, the primary photon beam attenuation for tissues in the human body, where the maximum difference was \(-0.9\%\) for bone. The EPL model also provides a good estimate, to within 1.5%, of the primary photon beam attenuation in lung, adipose, tissue, muscle, and cartilage, where the EPL values for these materials are systematically higher than eEPL. However, for bone and air, EPL incorrectly estimates primary photon beam attenuation due to the significantly different atomic composition of these materials relative to water; therefore the EPL model will incorrectly predict the scattered photon fluence in this medium.

### Table III

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho_{\text{med}}^{-1} \left( \frac{l_{\text{H}<em>2\text{O}} - l</em>{\text{med}}}{l_{\text{med}}} \right) \times 100 ) (EPL)</th>
<th>( \rho_{\text{med}}^{-1} \left( \frac{l_{\text{H}<em>2\text{O}} - l</em>{\text{med}}}{l_{\text{med}}} \right) \times 100 ) (eEPL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>10.0</td>
<td>-0.1</td>
</tr>
<tr>
<td>Lung</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Adipose</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Tissue</td>
<td>1.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Muscle</td>
<td>0.9</td>
<td>0.1</td>
</tr>
<tr>
<td>Cartilage</td>
<td>1.4</td>
<td>0.0</td>
</tr>
<tr>
<td>Bone</td>
<td>6.5</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

Fig. 5. The ratio of photon attenuation lengths in water relative to a material, \( \rho_{\text{med}}^{-1}(l_{\text{H}_2\text{O}}/l_{\text{med}}) \), is presented for water-equivalent plastics. Photon attenuation lengths in water, \( l_{\text{H}_2\text{O}} \), and med, \( l_{\text{med}} \), were calculated using attenuation coefficients from the XCOM photon cross-section database (Ref. 22), where the values were divided by the mass density and electron density of water and medium, respectively, for the EPL and the eEPL methods. The water-equivalent plastics should have very similar absorption and scattering properties as water and therefore the EPL heterogeneity correction should provide a good estimate of the photon attenuation length in water of these materials. However, as may be seen in Fig. 5(a), this is not the case. All ratios are systematically lower than 1.00. These differences are a consequence of the difference in the hydrogen content in the plastics relative to water. Polystyrene and RW3 are the two plastic materials with a lower hydrogen content.

2. Water-equivalent plastics

In Figs. 5(a) and 5(b), the ratio of photon attenuation lengths in water relative to a material, \( \rho_{\text{med}}^{-1}(l_{\text{H}_2\text{O}}/l_{\text{med}}) \), for water-equivalent plastics defined in Table II, are presented, where \( \rho_{\text{med}} \) is either the ratio of the mass density or the electron density of water relative to the medium, respectively, for the EPL and the eEPL methods. The photon attenuation lengths \( l_{\text{H}_2\text{O}} \) and \( l_{\text{med}} \) were calculated using attenuation coefficients from the XCOM photon cross-section database, where the values were divided, respectively, by the mass density and electron density of water and medium in order to allow a comparison with the EPL and eEPL models. The water-equivalent plastics should have very similar absorption and scattering properties as water and therefore the EPL heterogeneity correction should provide a good estimate of the photon attenuation length in water of these materials. However, as may be seen in Fig. 5(a), this is not the case. All ratios are systematically lower than 1.00. These differences are a consequence of the difference in the hydrogen content in the plastics relative to water. Polystyrene and RW3 are the two plastic materials with a lower hydrogen content.
content than water, and, as may be seen in Fig. 5(a), are the materials with the lowest value for the ratio \(\frac{H_{2O}}{\rho_{med}}\) and therefore have different scattering properties to water. Since the scattering properties of the water-equivalent plastics are very different from that of water, the EPL heterogeneity correction will provide a poor estimate of the photon attenuation length per energy bin with the aid of Fig. 4(c).

In the case of the eEPL model, a significantly better estimate is provided of \(\rho_{med}^{H_{2O}}\) than EPL. For energies between 100 keV and 6 MeV, eEPL provides a good estimate of the photon attenuation lengths by up to 3%, where the largest differences are observed for polystyrene and RW3. For energies above 6 MeV, eEPL overestimates the photon attenuation lengths by up to 3%, giving Z dependence.

The EPL heterogeneity correction can be extended to include pair production in the following extension to Eq. (8):

\[
\rho_{med}^{H_{2O}} = \rho_{med}^{H_{2O}}(1 + \alpha_1 \ln(E) E^{1/2})
\]

where the pair-production attenuation coefficient has been added to the numerator and denominator of Eq. (8), in square brackets. The dependence of the attenuation coefficients on photon energy, \(E\), and the medium atomic number, \(Z\), are given by \(\mu_{compton} = Z/A \times E^{-1/2}\) and \(\mu_{pp} = (Z/A + Z^2/A) \ln(E)\), respectively, for the Compton and pair-production interactions, with the Compton effect having a weak \(E^{-1/2}\) energy dependence. Pair production can occur in the electromagnetic field of the electron (giving Z dependence) or the nucleus (giving \(Z^2\) dependence); \(\langle Z\rangle = (1/\langle Z/A\rangle)\langle Z^2/A\rangle\), with \(f_i\) the fraction by weight of the chemical element \(i\) and \((Z/A)\), the atomic number to the atomic weight of chemical element \(i\); \(\alpha_{pp} = 1.775 \times 10^{-3}\) is a fitting parameter that is independent of material and photon energy and was obtained by fitting Eq. (9) to the photon attenuation lengths \(l_{med}\) of all the materials defined in Tables I and II.

In Figs. 6(a) and 6(b), the ratio of photon attenuation length in water, to that of a material, \(\frac{l_{med}(H_{2O})}{l_{med}^{med}}\), for the tissues defined in Table I and the water-equivalent plastics defined in Table II, \(l_{med}^{med}\) are presented. The photon attenuation lengths \(l_{med}^{H_{2O}}\) and \(l_{med}^{med}\) were calculated using attenuation coefficients from the XCOM photon cross-section database, where the values were divided by the effective electron density of water and med, respectively, in order to allow a comparison with the eEPL estimate using Eq. (9), where this effective electron density is defined by

\[
\rho_{eff} = \rho_e (1 + \alpha_0 (1 + \langle Z \rangle) \ln(E) E^{1/2})
\]

The eEPL estimate provides an excellent estimate of the photon attenuation length, to within 1%, where only one single fitting parameter was required for all materials in Tables I and II and for energies between 100 keV and 20 MeV. The extended eEPL heterogeneity correction can now be used for materials with a high content of calcium (such as cortical bone, bone-equivalent plastics, etc.) or higher Z elements and/or for primary photon energies above...
The 2–3 cm of bone modeling of electron transport in convolution algorithms. Photon attenuation lengths in water, \( l_{H2O} \) and med \( l_{med} \) were calculated using attenuation coefficients from the XCOM photon cross-section database (Ref. 22), where the values were divided by the effective electron-density of water, \( \rho_{eff \ H2O} \) and med. \( \rho_{eff \ med} \) (which account for pair production), respectively, in order to allow a comparison with the eEPL model for (a) tissues in the human body and (b) water-equivalent plastics as defined in Tables I and II. If the eEPL heterogeneity correction is a good estimated of the photon attenuation length in a media other than water, then this ratio should be 1.00, as represented by a dotted line.

10 MeV, where the pair-production process can no longer be neglected in calculating the photon attenuation lengths.

V. DISCUSSION

The systematically larger EPL ranges relative to \( l_{H2O} \) observed in Figs. 4(a) and 5(a), lead to systematically higher estimates of the primary photon fluence in these media relative to water. This was also observed for the monoenergetic photon beam in Figs. 2(a) and 3(a). These higher estimates of the primary photon fluence are a consequence of the different hydrogen content between the media in Tables I and II and water. A higher estimate of the primary photon fluence will lead to a higher estimate of the primary dose in these media. This was also observed by Ahnesjö, who showed that the CC algorithm overestimated dose in lung. The incorrect lung dose can be attributed to the incorrect modeling of the photon beam attenuation through the rib cage (thickness of 2–3 cm in bone) and through the lung and the oversimplified modeling of electron transport in convolution algorithms. The 2–3 cm of bone (or 5 cm of water-equivalent thickness) can lead to differences between PB/CC and Monte Carlo dose predictions of approximately 2%–3% [as may be seen from Fig. 2(a) by comparing the 5 cm value between water and bone]. In addition to this, differences in the photon attenuation in lung can lead to differences of the order of 1%, as may be seen from Figs. 2 and 4. In the case of the eEPL method, a significantly better estimate is provided of the photon attenuation length for all materials, because differences in the hydrogen content of the various tissues and plastics relative to water are accounted for by the ratio of the electron density of the media relative to water.

The accuracy of eEPL heterogeneity correction was shown to be within 3% of photon attenuation length \( l_{H2O} \) for nearly all materials studied and for primary photon energies between 100 keV and 10 MeV, where differences depended on whether pair-production could be neglected. The eEPL method can be used to scale the photon beam dose distribution to media other than water, for 4, 6, and 10 MV radiotherapy treatment photon beams. For radiotherapy treatment beams with photon energies higher than 10 MeV (such as 15, 18, and 21 MV photon beams) and for tissues with high-Z chemical elements (such as calcium in cortical bone), the scaling of the dose distributions must take into account pair-production attenuation of the primary photons, and therefore the scaling of the dose distributions in media other than water becomes dependent on the primary photon energy. Attenuation of the primary photon beam by pair production can be accounted for by introducing a correction term to the electron density, as was done in Eq. (9), where the extended eEPL method was shown to estimate the photon attenuation length \( l_{H2O} \) to within 1% for all materials studied in Tables I and II and for energies between 100 keV and 20 MeV.

The electron density was shown to influence photon dose calculations, where the largest differences relative to mass-density predictions were observed for air and bone. Therefore, the largest differences are expected between PB or CC predictions and a Monte Carlo dose prediction (that accounts for electron-density variations in the patient), for treatment sites with bony structures or air cavities in the beam (such as...
head and neck tumors). For example, in the case of lung tumors, photon beams usually have approximately 2–3 cm of bone (or 5 cm of water-equivalent thickness) in the beam due to the rib cage; this would lead to differences between PB/CC and Monte Carlo dose predictions.

The electron-density effect on photon dose calculations was also shown to depend strongly on the energy of the photon beam used for treatment. Therefore, PB and CC dose estimates for energies equal to or higher than 10 MV become less accurate with increasing photon energy. This is a consequence of the increased probability of pair-production events occurring, which is not accounted for by the mass-density scaling methods used in the PB or CC algorithms, but is accounted for by the effective electron-density scaling method defined in the present work.

VI. CONCLUSIONS

Conventional photon dose calculation algorithms such as PB, CC, Batho, all perform EPL mass-density scaling of the dose distributions to media other than water. In this paper, we demonstrate that the important material property that should be taken into account by photon dose algorithms is the electron density, and not the mass density. We have shown that EPL mass-density scaling leads to an overestimation of the primary photon fluence for materials in the human body and water-equivalent plastic substitutes, where largest differences were observed for bone and air, respectively, 6%–7% and 10%. However, in the case of patients, differences are expected to be smaller due to the large complexity of a treatment plan and of the patient anatomy and atomic composition and of the smaller thickness of bone/air that incident photon beams of a treatment plan may have to traverse. Some incident beam directions may have 2–5 cm of bone (corresponding to regions of rib cage, femur, skull, or jaws) upstream to the tumor site, while others may have no bone in the field of view. An overestimate of the dose to tissues in the human body, such as the lung, have also been observed by Ahnesjö and others, and were attributed to approximations made in both the mass-density scaling and the modeling of the electron transport in convolution algorithms. The overestimated lung dose can be attributed to the incorrect modeling of the photon beam attenuation through the rib cage (thickness of 2–3 cm in bone) and through the lung and of the oversimplified modeling of the electron transport in bone and lung.

In the present study, the overestimation of the primary photon fluence, by the EPL method, was shown to be a consequence of the differences in the hydrogen content between the various media studied and water. A new scaling method, eEPL, was proposed based on electron-density scaling (instead of mass density) and was shown to accurately predict the primary photon fluence in media other than water to within 1%–2% for all the materials studied and for energies up to 5 MeV. For energies above 5 MeV, the accuracy of the electron-density scaling method was shown to depend on the photon energy, where for materials with a high content of calcium (such as bone, cortical bone), or higher-Z elements, or for primary photon energies above 10 MeV, the pair-production process can no longer be neglected in calculating the photon attenuation lengths. For these materials, the eEPL method was extended to account for pair-production attenuation of the primary photons, and therefore the scaling of the dose distributions in media other than water became dependent on the primary photon energy. The extended eEPL method was shown to estimate the photon attenuation length $l_{\text{H}_2\text{O}}$ to within 1% for materials studied and for energies from 100 keV to 20 MeV. Thus, the eEPL method proposed in the present paper can be used to scale the photon beam dose distribution to media other than water accurately to within 1%, for clinical radiotherapy photon beams with energies from 4 to 20 MV.

ACKNOWLEDGMENT

This work is supported by Cancer Research UK under Programme Grant No. SP2312/0201.

4Corresponding author. Electronic mail: jseco@partners.org
14ADAC-Pinnacle, “Pinnacle Physics Instructions for Use” Manual accompanying Pinnacle3 radiation therapy planning software obtainable from pros.support@philips.com (document called PhysicsInstructions.pdf), 2004.